UNIT I FUNDAMENTALS OF ANALOG COMMUNICATION

1.2 Principles of Amplitude Modulation

The modulating signal modulates amplitude, frequency or phase of the carrier according to its variations in amplitude. This results in amplitude, frequency or phase modulation. The frequency and phase modulation is also called angle modulation.

1.2.1 AM Envelope and Equation of AM Wave

In amplitude modulation, the amplitude of a carrier signal is varied according to variations in the amplitude of modulating signal. Fig. 1.2.1 shows the modulating signal in Fig. 1.2.1 (a) Fig. 1.2.1 (b) shows high frequency carrier and Fig. 1.2.1 (c) shows amplitude modulated signal.

In Fig. 1.2.1 (c), observe that the carrier frequency remains same, but its amplitude varies according to amplitude variations of the modulating signal.

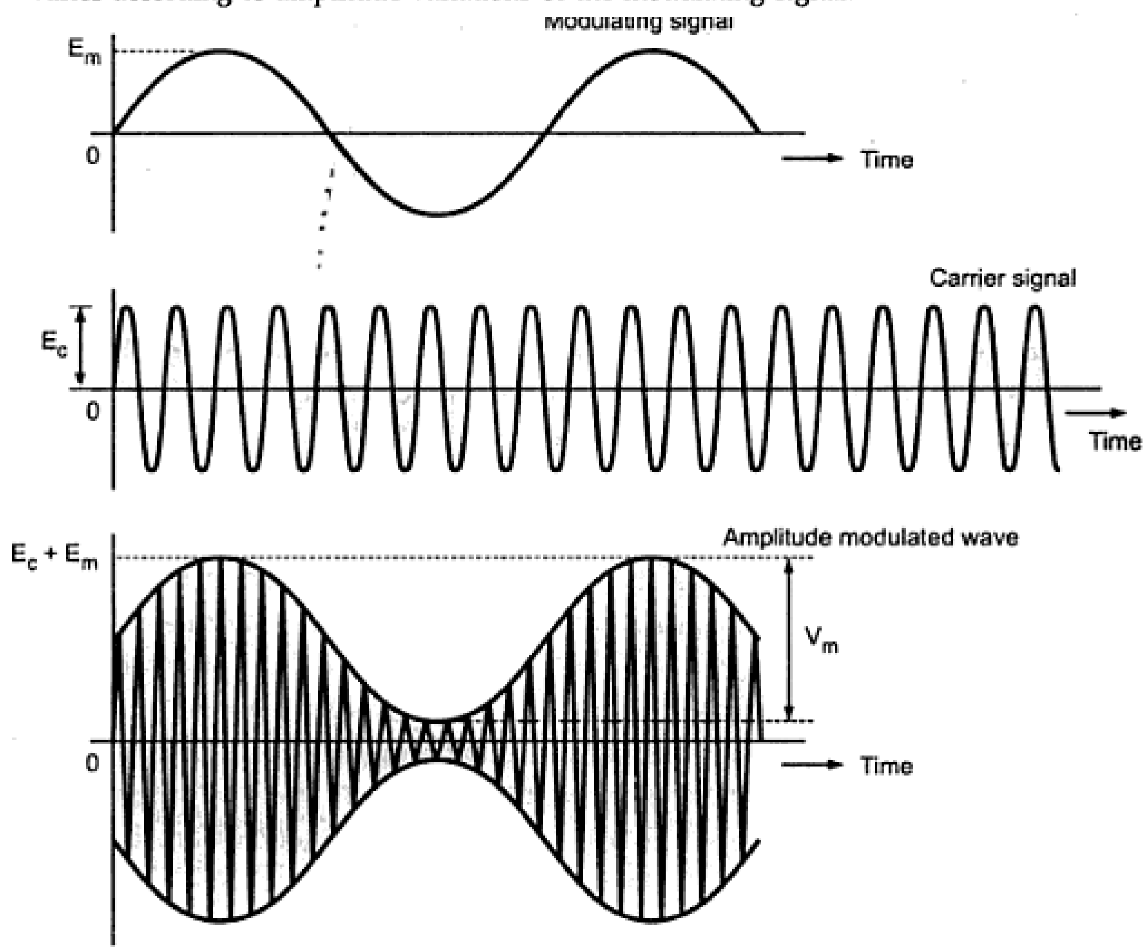


Fig. 1.2.1 (a) Sinusoidal modulating signal
(b) Sinusoidal high frequency carrier
(c) Amplitude modulated signal

Let us represent the modulating signal by e_m and it is given as,

$$e_m = E_m \sin \omega_m t \quad \text{... (1.2.1)}$$

and carrier signal can be represented by e_c as,

$$e_c = E_c \sin \omega_c t \quad \text{... (1.2.2)}$$

Here E_m is maximum amplitude of modulating signal

E_c is maximum amplitude of carrier signal

ω_m is frequency of modulating signal

andω_c is frequency of carrier signal.

Using the above mathematical expressions for modulating and carrier signals, we can create a new mathematical expression for the complete modulated wave. It is given as,

$$E_{AM} = E_c + e_m$$

$$= E_c + E_m \sin \omega_m t \quad \text{by putting } e_m \text{ from equation (1.2.1)}$$

$\therefore$ The instantaneous value of the amplitude modulated wave can be given as,

$$e_{AM} = E_{AM} \sin \theta$$

$$= E_{AM} \sin \omega_c t$$

$$\therefore \quad \boxed{e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t} \quad \text{... (1.2.3)}$$

This is an equation of AM wave.

1.2.2 Modulation Index and Percent Modulation

The ratio of maximum amplitude of modulating signal to maximum amplitude of carrier signal is called modulation index. i.e.,

$$\boxed{\text{Modulation index, } m = \frac{E_m}{E_c}} \quad \text{... (1.2.4)}$$

Value of E_m must be less than value of E_c to avoid any distortion in the modulated signal. Hence maximum value of modulation index will be equal to 1 when $E_m = E_c$. Minimum value will be zero. If modulation index is higher than 1, then it is called *over modulation*. Data is lost in such case. When modulation index is expressed in percentage, it is also called percentage modulation.

Calculation of modulation index from AM waveform :

Fig. 1.2.2 shows the AM waveform. This is also called time domain representation of AM signal.

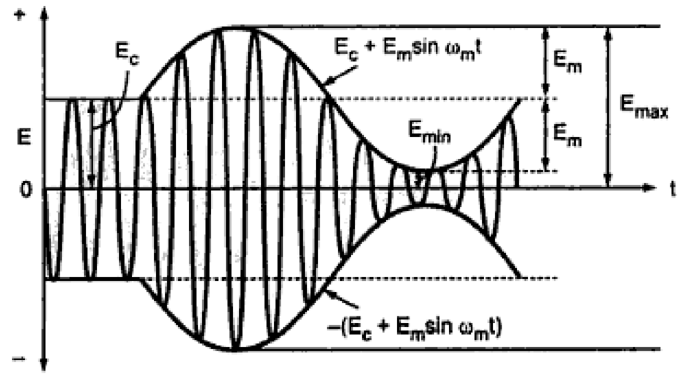


Fig. 1.2.2 AM wave

It is clear from the above signal that the modulating signal rides upon the carrier signal. From above figure we can write,

$$E_m = \frac{E_{max} - E_{min}}{2} \quad \text{... (1.2.5)}$$

and

$$E_c = E_{max} - E_m \quad \text{... (1.2.6)}$$

$$= E_{max} - \frac{E_{max} - E_{min}}{2} \text{ by putting for } E_m \text{ from equation (1.2.5)}$$

$$= \frac{E_{max} + E_{min}}{2} \quad \text{... (1.2.7)}$$

Taking the ratio of equation (1.2.5) and above equation,

$$m = \frac{E_m}{E_c} = \frac{\frac{E_{max} - E_{min}}{2}}{\frac{E_{max} + E_{min}}{2}}$$

$$\therefore \quad \boxed{m = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}} \quad \text{... (1.2.8)}$$

This equation gives the technique of calculating modulation index from AM wave.

1.2.3 Frequency Spectrum and Bandwidth

The modulated carrier has new signals at different frequencies, called side frequencies or sidebands. They occur above and below the carrier frequency.

i.e.

$$f_{USB} = f_c + f_m$$

$$f_{LSB} = f_c - f_m$$

Here f_c is carrier frequency and

f_m is modulating signal frequency

f_{LSB} is lower sideband frequency

Consider the expression of AM wave given by equation (1.2.3), i.e.,

$$e_{AM} = (E_c + E_m \sin \omega_m t) \sin \omega_c t \qquad \text{... (1.2.9)}$$

We know that $m = \frac{E_m}{E_c}$ from equation (1.2.4). Hence we have $E_m = m E_c$. Putting this value of E_m in above equation we get,

$$\begin{aligned} e_{AM} &= (E_c + m E_c \sin \omega_m t) \sin \omega_c t \\ &= E_c (1 + m \sin \omega_m t) \sin \omega_c t \\ &= E_c \sin \omega_c t + m E_c \sin \omega_m t \sin \omega_c t \qquad \text{... (1.2.10)} \end{aligned}$$

We know that $\sin(A) \sin(B) = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$. Applying this result to last term in above equation we get,

$$\begin{aligned} e_{AM} &= E_c \sin \omega_c t + \frac{m E_c}{2} \cos(\omega_c - \omega_m) t \\ &\quad - \frac{m E_c}{2} \cos(\omega_c + \omega_m) t \qquad \text{... (1.2.11)} \end{aligned}$$

In the above equation, the first term represents unmodulated carrier, the second term represents lower sideband and last term represents upper sideband. Note that $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$. Hence above equation can also be written as,

$$\begin{aligned} e_{AM} &= E_c \sin 2\pi f_c t + \frac{m E_c}{2} \cos 2\pi (f_c - f_m) t \\ &\quad - \frac{m E_c}{2} \cos 2\pi (f_c + f_m) t \qquad \text{... (1.2.12)} \\ &= E_c \sin 2\pi f_c t + \frac{m E_c}{2} \cos 2\pi f_{LSB} t + \frac{m E_c}{2} \cos 2\pi f_{USB} t \qquad \text{... (1.2.13)} \end{aligned}$$

From this equation we can prepare the frequency spectrum of AM wave as shown below in Fig. 1.2.3.

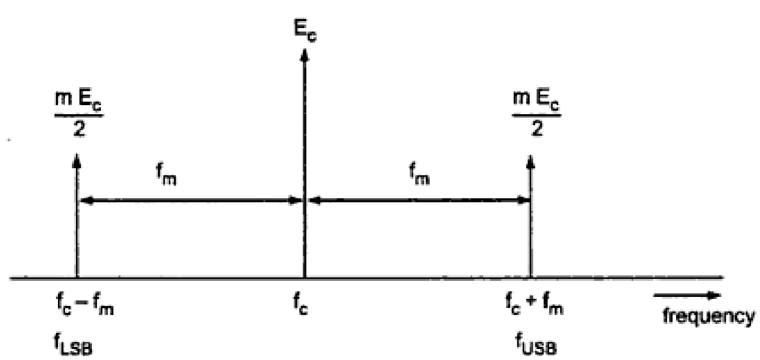


Fig. 1.2.3 Frequency domain representation of AM wave

This contains full carrier and both the sidebands, hence it is also called Double Sideband Full Carrier (DSBFC) system. We will be discussing this system, its modulation circuits and transmitters next, in this section.

We know that bandwidth of the signal can be obtained by taking the difference between highest and lowest frequencies. From above figure we can obtain bandwidth of AM wave as,

$$BW = f_{USB} - f_{LSB}$$

$$= (f_c + f_m) - (f_c - f_m)$$

$$\therefore \quad \boxed{BW = 2 f_m} \quad \text{... (1.2.14)}$$

Thus bandwidth of AM signal is twice of the maximum frequency of modulating signal.

Example 1.2.1 : *Calculate the modulation index and percentage modulation if instantaneous voltages of modulating signal and carrier are $40 \sin \omega_m t$ and $50 \sin \omega_c t$, respectively.*

Solution : From the given instantaneous equation we have,

$$E_m = 40 \quad \text{and} \quad E_c = 50$$

Hence modulation index will be,

$$m = \frac{E_m}{E_c} = \frac{40}{50} = 0.8$$

or

$$\% \text{ modulation} = m \times 100$$

$$= 0.8 \times 100 = 80\%$$

Example 1.2.2 : *The tuned circuit of the oscillator in a simple AM transmitter employs a $40 \mu H$ coil and 12 nF capacitor. If the oscillator output is modulated by audio frequency of 5 kHz, what are the lower and upper sideband frequencies and the bandwidth required to transmit this AM wave ?*

Solution : The frequency of the LC oscillator is given as,

$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{40 \times 10^{-6} \times 12 \times 10^{-9}}}$$

$$= 230 \text{ kHz}$$

The modulating frequency is $f_m = 5 \, kHz$

$$\therefore \quad f_{USB} = f_c + f_m = 230 + 5 = 235 \, kHz$$

and

$$f_{LSB} = f_c - f_m = 230 - 5 = 225 \, kHz$$

We know that bandwidth of AM wave is,

$$BW = 2 f_m$$

$$= 2 \times 5 \, kHz = 10 \, kHz$$

1.2.4 AM Power Distribution

May/ June - 2006

We know that AM signal has three components : Unmodulated carrier, lower sideband and upper sideband. Hence total power of AM wave is the sum of carrier power P_c and powers in the two sidebands P_{USB} and P_{LSB}. i.e.,

$$P_{Total} = P_c + P_{USB} + P_{LSB}$$

$$= \frac{E_{carr}^2}{R} + \frac{E_{LSB}^2}{R} + \frac{E_{USB}^2}{R} \quad \text{... (1.2.15)}$$

Here all the three voltages are rms values and R is characteristic impedence of antenna in which the power is dissipated. The carrier power is,

$$P_c = \frac{E_{carr}^2}{R} = \frac{\left(E_c / \sqrt{2}\right)^2}{R}$$

$$= \frac{E_c^2}{2R} \quad \text{... (1.2.16)}$$

The power of upper and lower sidebands is same. i.e.,

$$P_{LSB} = P_{USB} = \frac{E_{SB}^2}{R}$$ Here E_{SB} is rms voltage of sidebands.

From equation (1.2.13) we know that the peak amplitude of both the sidebands is $\frac{m E_c}{2}$. Hence,

$$E_{SB} = \frac{m E_c / 2}{\sqrt{2}}$$

$$\therefore \quad P_{LSB} = P_{USB} = \left(\frac{m E_c / 2}{\sqrt{2}}\right)^2 \times \frac{1}{R}$$

$$= \frac{m^2 E_c^2}{8R} \quad \text{... (1.2.17)}$$

Hence the total power (equation 1.2.15) becomes,

$$P_{Total} = \frac{E_c^2}{2R} + \frac{m^2 E_c^2}{8R} + \frac{m^2 E_c^2}{8R}$$

$$= \frac{E_c^2}{2R}\left[1 + \frac{m^2}{4} + \frac{m^2}{4}\right]$$

$$\therefore \quad \boxed{P_{Total} = P_c\left(1 + \frac{m^2}{2}\right)} \quad \text{... (1.2.19)}$$

$$\frac{P_{Total}}{P_c} = 1 + \frac{m^2}{2} \quad \text{... (1.2.20)}$$

This equation relates total power of AM wave to carrier power. Maximum value of modulation index, $m = 1$ to avoid distortion. At this value of modulation index, $P_{Total} = 1.5\, P_c$. From above equation we have,

$$\frac{m^2}{2} = \frac{P_{Total}}{P_c} - 1$$

$$\therefore \qquad m = \sqrt{2\left(\frac{P_{total}}{P_c} - 1\right)} \qquad \text{... (1.2.21)}$$

Example 1.2.3 : *An audio frequency signal $10 \sin 2\pi \times 500\, t$ is used to amplitude modulate a carrier of $50 \sin 2\pi \times 10^5\, t$. Calculate*

(i) Modulation index

(ii) Sideband frequencies

(iii) Amplitude of each sideband frequencies

(iv) Bandwidth required

(v) Total power delivered to the load of $600\,\Omega$.

Solution : (i) The given modulating signal is $e_m = 10 \sin 2\pi \times 500\, t$. Hence, $E_m = 10$. The given carrier signal is $e_c = 50 \sin 2\pi \times 10^5\, t$, hence, $E_c = 50$. Therefore modulation index will be,

$$m = \frac{E_m}{E_c} = \frac{10}{50} = 0.2 \quad \text{or} \quad 20\%$$

(ii) From the given equations,

$$\omega_m = 2\pi \times 500, \qquad \text{Hence } f_m = 500\, Hz$$

And $\omega_c = 2\pi \times 10^5$, Hence $f_c = 10^5\, Hz$ or 100 kHz

We know that $f_{USB} = f_c + f_m = 100\, kHz + 500\, Hz = 100.5\, kHz$

and $P_{LSB} = f_c - f_m = 100\, kHz - 500\, Hz = 99.5\, kHz$.

(iii) From equation (1.2.13) we know that the amplitudes of upper and lower sidebands is given as,

$$\text{Amplitude of upper and lower sidebands} = \frac{m E_c}{2} = \frac{0.2 \times 50}{2} = 5V$$

(iv) Bandwidth of AM wave is given by equation (1.2.10) as,

$$BW \text{ of } AM = 2 f_m = 2 \times 500\, Hz = 1\, kHz$$

(v) Total power delivered to the load is given by equation (1.2.18) as

$$P_{total} = \frac{E_c^2}{2R}\left(1 + \frac{m^2}{2}\right) = \frac{50^2}{2 \times 600}\left(1 + \frac{(0.2)^2}{2}\right)$$

$$\therefore \qquad = 2.125 \text{ watts}$$

Example 1.2.4 : *A 400 W carrier is modulated to a depth of 80% calculate the total power in the modulated wave.*

Solution : Here carrier power $P_c = 400\, W$ and $m = 0.8$.

From equation (1.2.19) total power is,

$$P_{total} = P_c\left(1+\frac{m^2}{2}\right) = 400\left(1+\frac{(0.8)^2}{2}\right)$$

$$= 528\,W$$

Example 1.2.5 : *A broadcast transmitter radiates 20 kW when the modulation percentage is 75. Calculate carrier power and power of each sideband.*

Solution : Here total power $P_{total} = 20{,}000\,W$ and $m = 0.75$

From equation (1.2.19) we have $P_{total} = P_c\left(1+\frac{m^2}{2}\right)$

$$\therefore \quad 20{,}000 = P_c\left(1+\frac{(0.75)^2}{2}\right)$$

$$\therefore \quad P_c = 15.6\,kW$$

We know that $P_{total} = P_c\left(1+\frac{m^2}{2}\right) = P_c + P_c\,\frac{m^2}{2}$

The second term in above equation represents total sideband power. Hence power of one sideband will be,

$$P_{SB} = \left(P_c\,\frac{m^2}{2}\right)\times\frac{1}{2}$$

$$= 15.6\times\frac{(0.75)^2}{2}\times\frac{1}{2}$$

$$= 2.2\,kW$$

Thus $P_{USB} = P_{LSB} = 2.2\,kW$

2.1 Angle Modulation

2.1.1 Definition

We know that amplitude, frequency or phase of the carrier can be varied by the modulating signal. Amplitude is varied in AM. *When frequency or phase of the carrier is varied by the modulating signal, then it is called angle modulation.* There are two types of angle modulation.

1. Frequency Modulation : When frequency of the carrier varies as per amplitude variations of modulating signal, then it is called Frequency Modulation (FM). Amplitude of the modulated carrier remains constant.

2. Phase Modulation : When phase of the carrier varies as per amplitude variations of modulating signal, then it is called Phase Modulation (PM). Amplitude of the modulated carrier remains constant.

The angle modulated wave is mathematically expressed as,

$$e(t) = E_c \sin[\omega_c t + \theta(t)]$$

Here $e(t)$ is angle modulated wave

E_c is peak amplitude of the carrier

ω_c carrier frequency

$\theta(t)$ instantaneous phase deviation

The phase deviation takes place in FM as well as PM. Hence phase is direct function of modulating signal. i.e.,

$$\theta(t) \propto e_m(t)$$

Here $e_m(t)$ is the modulating signal.

2.1.3 FM and PM Waveforms

Fig. 2.1.1 shows the waveforms of FM and PM.

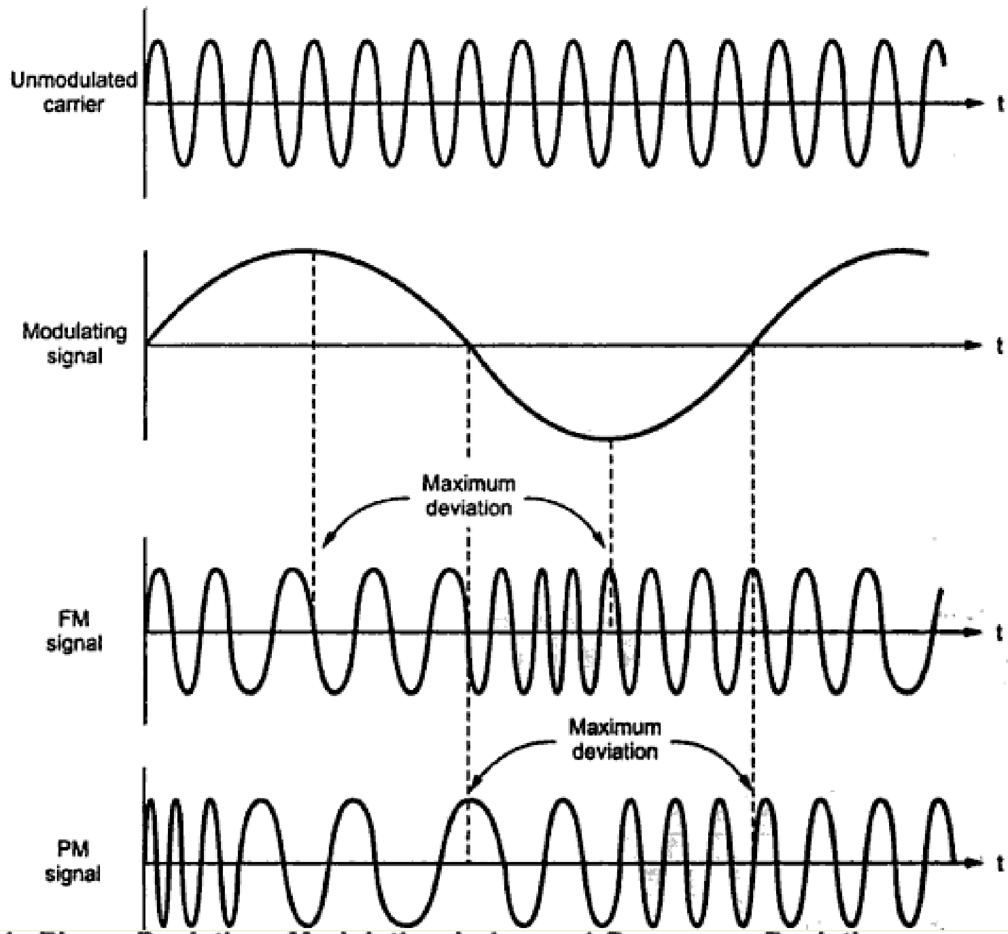
In this figure following observations can be noted :

(i) For FM signal, the maximum frequency deviation takes place when modulating signal is at positive and negative peaks.

(ii) For PM signal the maximum frequency deviation takes place near zero crossings of the modulating signal.

(iii) Both FM and PM waveforms are identical except the phase shift.

(iv) From modulated waveform it is difficult to know, whether the modulation is FM or PM.



2.1.4 Phase Deviation, Modulation Index and Frequency Deviation

The FM signal, in general is expressed as,

$$e_{FM}(t) = E_c \sin[\omega_c t + m \sin \omega_m t] \quad \text{... (2.1.12)}$$

And the PM signal, in general is expressed as,

$$e_{PM}(t) = E_c \sin[\omega_c t + m \cos \omega_m (t)] \quad \text{... (2.1.13)}$$

In both the above equations, the term 'm' is called *modulation index*. Note that the term $m \sin \omega_m t$ in equation 2.1.12 and $m \cos \omega_m t$ in equation 2.1.13 indicates instantaneous phase deviation $\theta(t)$. Hence 'm' also indicates *maximum phase deviation*. In other words, modulation index can also be defined as maximum phase deviation.

Modulation index for PM :

Comparing equation 2.1.13 and equation 2.1.11, we find that,

$$\textbf{Modulation index in PM : } m = k E_m \text{ rad} \quad \text{... (2.1.14)}$$

Thus modulation index of PM signal is directly proportional to peak modulating voltage. And it's unit is radians.

Modulation index for FM :

Comparing equation 2.1.12 and equation 2.1.10 we find that,

$$m = \frac{k_1 E_m}{\omega_m} \qquad \text{... (2.1.15)}$$

Thus modulation index of FM is directly proportional to peak modulating voltage, but inversely proportional to modulating signal frequency.

Since $\omega_m = 2\pi f_m$ above equation becomes,

$$m = \frac{k_1 E_m}{2\pi f_m}$$

Here $\frac{k_1 E_m}{2\pi}$ is called *frequency deviation*. It is denoted by δ and its unit is Hz, i.e.,

$$\textbf{Modulation index in FM :} \quad m = \frac{\delta}{f_m} = \frac{\text{Maximum frequency deviation}}{\text{Modulating frequency}} \qquad \text{... (2.1.16)}$$

Thus modulation index of FM is unitless ratio. From above equation and equation 2.1.14, note that the modulation index is differently defined for FM and PM signals.

Percentage modulation :

For angle modulation, the percentage modulation is given as the ratio of actual frequency deviation to maximum allowable frequency deviation. i.e.,

$$\%\ \text{modulation} = \frac{\text{Actual frequency deviation}}{\text{Maximum allowable frequency deviation}} \qquad \text{... (2.1.17)}$$

2.1.5 Frequency Spectrum of Angle Modulated Waves

We know that AM contains only two sidebands per modulating frequency. But angle modulated signal contains large number of sidebands depending upon the modulation index. Since FM and PM have identical modulated waveforms, their frequency content is same. Consider the PM equation for spectrum analysis,

$$e(t) = E_c \sin[\omega_c t + m \cos \omega_m t]$$

Using Bessel functions, this equation can be expanded as,

$$\begin{aligned} e(t) = E_c \{ & J_0 \sin \omega_c t \\ & + J_1 [\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t] \\ & + J_2 [\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t] \\ & + J_3 [\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t] \\ & + J_4 [\sin(\omega_c + 4\omega_m)t - \sin(\omega_c - 4\omega_m)t] + \ldots \} \end{aligned}$$

Here $J_0, J_1, J_2 \ldots$ are the Bessel functions. The values of Bessel functions depend upon modulation index m. They are listed in Table 2.1.1

x	n or Order																
(m)	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}	J_{15}	J_{16}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	—	—	—	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	0.01	—	—	—	—
9.0	−0.09	0.24	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.30	0.21	0.12	0.06	0.03		—	—	—
10.0	−0.25	0.04	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.31	0.29	0.20	0.12	0.06	0.03	0.01	—	—
12.0	0.05	−0.22	−0.08	0.20	0.18	−0.07	−0.24	−0.17	0.05	0.23	0.30	0.27	0.20	0.12	0.07	0.03	0.01
15.0	−0.01	0.21	0.04	−0.19	−0.12	0.13	0.21	0.03	−0.17	−0.22	−0.09	0.10	0.24	0.28	0.25	0.18	0.12

It is clear from the above discussion that, angle modulated signal has infinite number of sidebands as well as carrier in the output. The sidebands are separated from the carrier by $f_m, 2f_m, 3f_m$..... etc. The frequency separation between successive sidebands is f_m. All the sidebands are symmetric around carrier frequency. The amplitudes of the sidebands are $E_c J_0$, $E_c J_1$, $E_c J_2$, $E_c J_3$, $E_c J_4$, ... and so on.

2.1.6 Bandwidth Requirement

The bandwidth requirement of angle modulated waveforms can be obtained depending upon modulation index. The modulation index can be classified as low (less than 1), medium (1 to 10) and high (greater than 10). The low index systems are called *narrowband FM*. For such systems the frequency spectrum resembles AM. Hence minimum bandwidth is given as,

$$BW = 2 f_m \ Hz \qquad \text{... (2.1.19)}$$

For high index modulation, the minimum bandwidth is given as,

$$BW = 2\delta \qquad \text{... (2.1.20)}$$

The bandwidth can also be obtained using bessel table. i.e.,

$$BW = 2 n f_m \qquad \text{... (2.1.21)}$$

Here 'n' is the number of significant sidebands obtained from bessel table.

Carson's rule :

Carson's rule gives approximate minimum bandwidth of angle modulated signal as,

$$\boxed{BW = 2\left[\delta + f_{m(\max)}\right] Hz} \qquad \text{... (2.1.22)}$$

Here $f_{m(\max)}$ is the maximum modulating frequency. As per Carson's rule, the bandwidth accomodates almost 98% of the total transmitted power.

UNIT II DIGITAL COMMUNICATION

INTRODUCTION:

2.2 Shannon Limit for Information Capacity

Definition of information capacity : It is an ability of the system to carry number of independent symbols in a given unit of time. The capacity is expressed in bits per second.

Shannon's limit for information capacity

The capacity of a white bandlimited gaussian channel is given as,

$$C = B\log_2\left(1+\frac{S}{N}\right) \text{ bits/sec} \qquad \ldots(2.2.1)$$

Here C is the channel capacity

B is the channel bandwidth

$\frac{S}{N}$ is the signal to noise power ratio.

2.3 Digital Amplitude Modulation or Amplitude Shift Keying

The amplitude shift keying is also called on-off keying (OOK). This is the simplest digital modulation technique. The binary input data is converted to unipolar NRZ signal. A product modulator takes this NRZ signal and carrier signal. The output of the product modulator is the ASK signal, which can be expressed mathematically as,

$$v(t) = d\sin(2\pi f_c t) \qquad \ldots(2.3.1)$$

Here f_c is the carrier frequency

and d is the data bit, which is either 1 or 0.

Fig. 2.3.1 (a) shows the block diagram of the ASK modulator. The binary data sequence 'd' is given to the NRZ level encoder. This NRZ level encoder converts the input binary sequence to the signal suitable for product modulator. The product modulator also accepts a sinusoidal carrier of frequency f_c. The output of the product modulator is passed through a bandpass filter for bandwidth limiting. The output of the bandpass filter is the ASK signal. This signal and other waveforms are shown in Fig. 2.3.1 (b). Observe that the ASK signal has on-off nature. In equation 2.3.1 when $d=0$, $v(t)=0$; i.e. no ASK signal. And when $d=1$, $d=\sin(2\pi f_c t)$. The ASK is very sensitive to noise. It is used for very low bit rates less than around 100 bps. The only advantage of ASK is that it is very simple to implement.

Baud rate

For ASK, the ASK waveform is changed at the bit rate. Hence Baud rate is given as,

$$\text{Baud rate} = f_b \qquad \ldots(2.3.2)$$

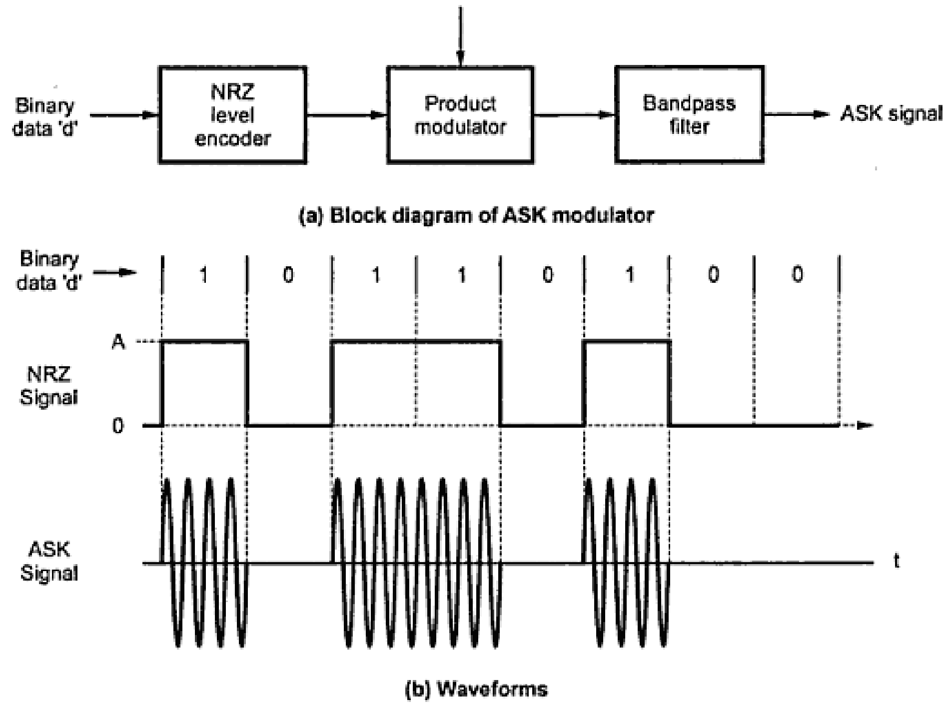
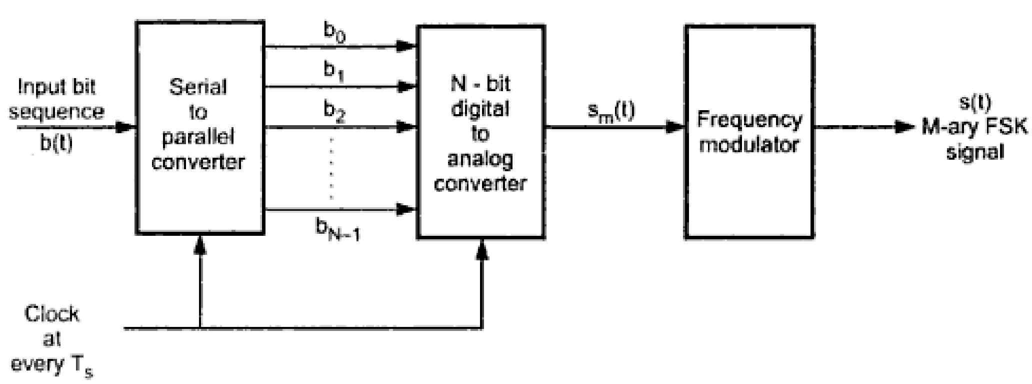


Fig. 2.3.1 Amplitude shift keying (ASK)

FREQUENCY SHIFT KEYING:

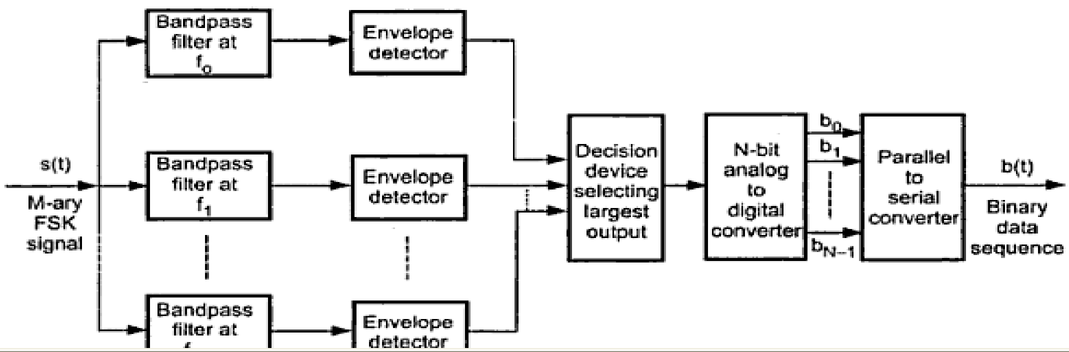
3.8.1.1 Transmitter

Fig. 3.8.1 shows the M-ary FSK transmitter. The 'N' successive bits are presented in parallel to digital to analog converter. These 'N' bits forms a symbol at the output of digital to analog converter. There will be total $2^N = M$ possible symbols. The symbol is presented every $T_s = NT_b$ period. The output of digital to analog converter is given to a frequency modulator. Thus depending upon the value of symbol, the frequency modulator generates the output frequency. For every symbol, the frequency modulator produces different frequency output. This particular frequency signal remains at the output for one symbol duration. Thus for 'M' symbols, there are 'M' frequency signals at the output of modulator. Thus the transmitted frequencies are $f_0, f_1, f_2, \ldots f_{M-1}$ depending upon the input symbol to the modulator.



3.8.1.2 Receiver

Fig. 3.8.2 shows block diagram of M-ary FSK receiver. It is the extension of BFSK receiver of Fig. 3.8.1. The M-ary FSM signal is given to the set of 'M' bandpass filters. The center frequencies of those filters are $f_0, f_1, f_2, \ldots f_{M-1}$. These filters pass their particular frequency and alternate others. The envelope detectors outputs are applied to a decision device. The decision device produces its output depending upon the highest input. Depending upon the particular symbol, only one envelope detector will have higher output. The outputs of other detectors will be very low. The output of the decision device is given to 'N' bit analog to digital converter. The analog to digital converter output is the 'N' bit symbol in parallel. These bits are then converted to serial bit stream by parallel to serial converter. In some cases the bits appear in parallel. Then there is no need to use serial to parallel and parallel to serial converters.



3.8.2 Power Spectral Density and Bandwidth of M-ary FSK

We know that for M symbol $f_0, f_1, f_2 \cdots f_{m-1}$ frequencies are used for transmission. The probability of error is minimized by selecting those frequencies such that transmitted signals are mutually orthogonal. If those frequencies are selected as successive even harmonics of symbol frequency f_s, then transmitted signals will be orthogonal.

Let's say that the lowest carrier frequency f_0 is the k^{th} harmonic of symbol frequency i.e.,

$$f_0 = kf_s \qquad \ldots (3.8.1)$$

Then the other frequencies will be,

$$f_1 = (k+2)f_s, f_2 = (k+4)f_s \ldots \text{etc} \qquad \ldots (3.8.2)$$

Thus every carrier frequency is separated by $2f_s$ from its nearest carriers. Fig.3.7.2 shows the power spectral density of BFSK (for two symbol FSK). In this plot the two symbol frequencies f_L and f_H are separated by $2f_s$ (Here $f_s = f_b$ for BFSK). The same principle of BFSK is extended to M-ary FSK. That is M-carriers are added with separation of $2f_s$ between the carrriers (Note here that f_s is symbol frequency and not f_b). Therefore power spectral density for M-ary FSK will be simply extension of BFSK. Fig. 3.8.3 shows the power spectral density of M-ary FSK.

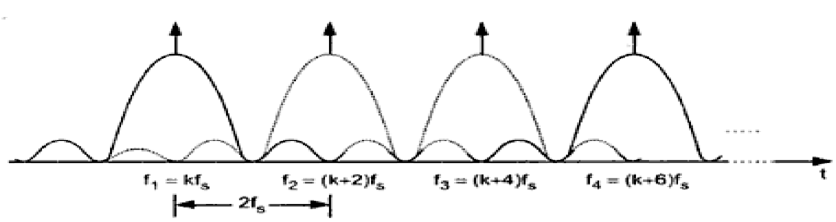


Fig. 3.8.3 Power spectral density M-ary FSK

Bandwidth of M-ary FSK :

From Fig. 3.8.3 it is clear that the width of one main lobe is $2f_s$. If there are M-symbols, then power spectral density spectrum will have M lobes. Therefore bandwidth of the system for M-symbols will be

$$BW = M \times (2f_s)$$

$$= 2Mf_s \quad \text{... (3.8.3)}$$

We know that $2^N = M$ and $f_s = \frac{f_b}{N}$ we can write the above equations,

$$BW = 2 \cdot 2^N \cdot \frac{f_b}{N} \quad \text{... (3.8.4)}$$

$$= \frac{2^{N+1} f_b}{N} \quad \text{... (3.8.5)}$$

4.2 Binary Phase Shift Keying (BPSK)

4.2.1 Principle of BPSK

- In binary phase shift keying (BPSK), binary symbol '1' and '0' modulate the phase of the carrier. Let the carrier be,

$$s(t) = A \cos(2\pi f_0 t) \quad \text{... (4.2.1)}$$

'A' represents peak value of sinusoidal carrier. In the standard 1Ω load register, the power dissipated will be,

$$P = \frac{1}{2} A^2$$

$$\therefore \quad A = \sqrt{2P} \quad \text{... (4.2.2)}$$

- When the symbol is changed, then the phase of the carrier is changed by 180 degrees (π radians).
- Consider for example,

Symbol '1' $\Rightarrow s_1(t) = \sqrt{2P} \cos(2\pi f_0 t)$ ... (4.2.3)

if next symbol is '0' then,

Symbol '0' $\Rightarrow s_2(t) = \sqrt{2P} \cos(2\pi f_0 t + \pi)$ ... (4.2.4)

Since $\cos(\theta + \pi) = -\cos\theta$, we can write above equation as,

$$s_2(t) = -\sqrt{2P} \cos(2\pi f_0 t) \quad \text{... (4.2.5)}$$

With the above equation we can define BPSK signal combinely as,

$$\boxed{s(t) = b(t)\sqrt{2P} \cos(2\pi f_0 t)} \quad \text{... (4.2.6)}$$

Here $b(t) = +1$ when binary '1' is to be transmitted

$= -1$ when binary '0' is to be transmitted

4.2.2 Graphical Representation of BPSK Signal

Fig. 4.2.1 shows binary signal and its equivalent signal $b(t)$.

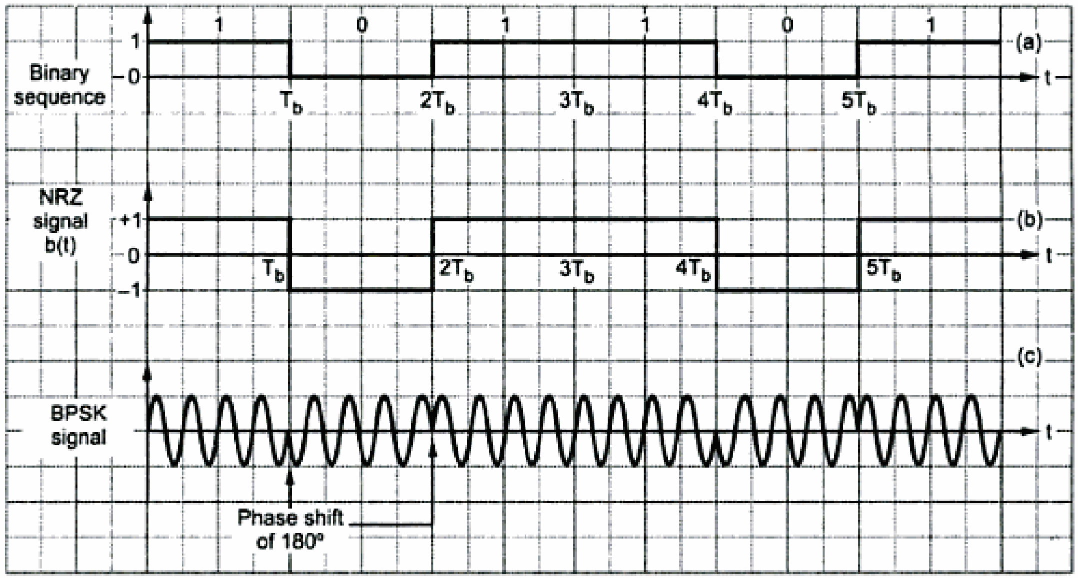


Fig. 4.2.1 (a) Binary sequence
(b) Its equivalent bipolar signal $b(t)$
(c) BPSK signal

As can be seen from Fig. 4.2.1 (b), the signal $b(t)$ is NRZ bipolar signal. This signal directly modulates carrier $\cos(2\pi f_0 t)$.

4.2.3 Generation and Reception of BPSK Signal

4.2.3.1 Generation of BPSK Signal

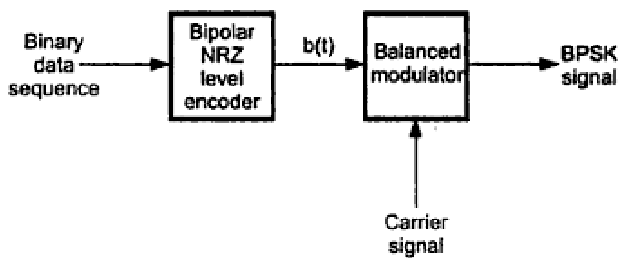


Fig. 4.2.2 BPSK generation scheme

- The BPSK signal can be generated by applying carrier signal to the balanced modulator.
- The baseband signal $b(t)$ is applied as a modulating signal to the balanced modulator. Fig. 4.2.2 shows the block diagram of BPSK signal generator.
- The NRZ level encoder converts the binary data sequence into bipolar NRZ signal.

4.2.3.2 Reception of BPSK Signal

Fig. 4.2.3 shows the block diagram of the scheme to recover baseband signal from BPSK signal. The transmitted BPSK signal is,

$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$$

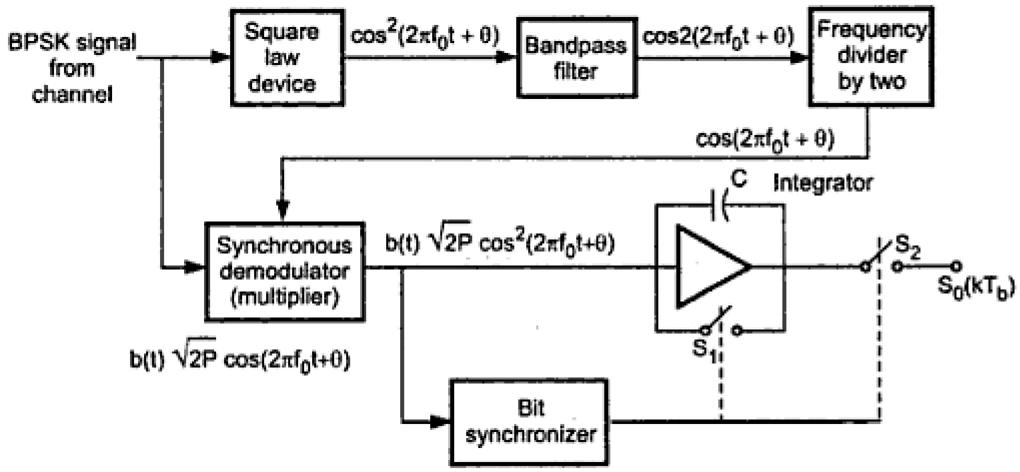


Fig. 4.2.3 Reception BPSK scheme

Operation of the receiver

1) **Phase shift in received signal** : This signal undergoes the phase change depending upon the time delay from transmitter to receiver. This phase change is normally fixed phase shift in the transmitted signal. Let the phase shift be θ. Therefore the signal at the input of the receiver is,

$$s(t) = b(t)\sqrt{2P}\,cos(2\pi f_0\,t+\theta) \qquad \text{... (4.2.7)}$$

2) **Square law device** : Now from this received signal, a carrier is separated since this is coherent detection. As shown in the figure, the received signal is passed through a square law device. At the output of the square law device the signal will be,

$$cos^2(2\pi f_0\,t+\theta)$$

Note here that we have neglected the amplitude, because we are only interested in the carrier of the signal.

We know that,

$$cos^2\theta = \frac{1+cos\,2\theta}{2}$$

$$\therefore \quad cos^2(2\pi f_0\,t+\theta) = \frac{1+cos\,2(2\pi f_0\,t+\theta)}{2}$$

4.2.4 Spectrum of BPSK Signals

Step 1 : Fourier transform of basic NRZ pulse.

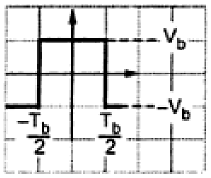


Fig. 4.2.4 NRZ pulse

We know that the waveform $b(t)$ is NRZ bipolar waveform. In this waveform there are rectangular pulses of amplitude $\pm V_b$. If we say that each pulse is $\pm \frac{T_b}{2}$ around its center as shown in Fig. 4.2.4. then it becomes easy to find fourier transform of such pulse. The fourier transform of this type of pulse is given as,

$$X(f) = V_b T_b \frac{sin(\pi f T_b)}{(\pi f T_b)} \quad \text{By standard relations} \qquad \text{... (4.2.10)}$$

Step 2 : PSD of NRZ pulse.

For large number of such positive and negative pulses the power spectral density $S(f)$ is given as

$$S(f) = \frac{\overline{|X(f)|^2}}{T_s} \qquad \text{... (4.2.11)}$$

Here $\overline{X(f)}$ denotes average value of $X(f)$ due to all the pulses in $b(t)$. And T_s is symbol duration. Putting value of $X(f)$ from equation 4.2.10 in equation 4.2.11 we get,

Plot of PSD

- Equation 4.2.12 gives power spectral density of the NRZ waveform. For one rectangular pulse, the shape of $S(f)$ will be a sinc pulse as given by equation 4.2.12. Fig. 4.2.5 shows the plot of magnitude of $S(f)$.

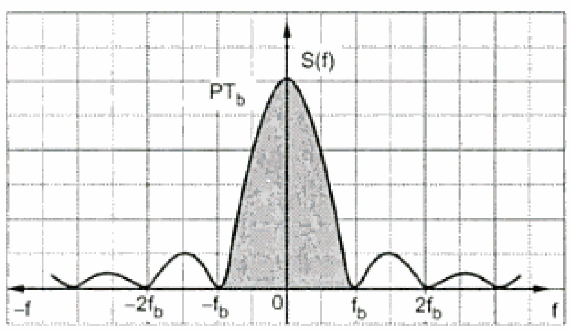


Fig. 4.2.5 Plot of power spectral density of NRZ baseband signal

Above figure shows that the main lobe ranges from $-f_b$ to $+f_b$. Here $f_b = \frac{1}{T_b}$.

Since we have taken $\pm V_b = \pm\sqrt{P}$ in equation 4.2.12, the peak value of the main lobe is PT_b.

- Now let us consider the power spectral density of BPSK signal given by equation 4.2.13. Fig. 4.2.6 shows the plot of this equation. The figure thus clearly shows that there are two lobes ; one at f_0 and other at $-f_0$. The same spectrum of Fig. 4.2.5 is placed at $+f_0$ and $-f_0$. But the amplitudes of main lobes are $\frac{PT_b}{2}$ in Fig. 4.2.6.

4.2.6 Bandwidth of BPSK Signal

The spectrum of the BPSK signal is centered around the carrier frequency f_0.

If $f_b = \frac{1}{T_b}$, then for BPSK the maximum frequency in the baseband signal will be f_b see Fig. 4.2.6. In this figure the main lobe is centered around carrier frequency f_0 and extends from $f_0 - f_b$ to $f_0 + f_b$. Therefore Bandwidth of BPSK signal is,

$$BW = \text{Highest frequency} - \text{Lowest frequency in the main lobe}$$

$$= f_0 + f_b - (f_0 - f_b)$$

$$\therefore \quad \boxed{BW = 2f_b} \quad \text{... (4.2.21)}$$

Thus the minimum bandwidth of BPSK signal is equal to twice of the highest frequency contained in baseband signal.

4.4 Quadrature Phase Shift Keying (QPSK)

Principle

- In communication systems we know that there are two main resources, i.e. transmission power and the channel bandwidth. The channel bandwidth depends upon the bit rate or signalling rate f_b. In digital bandpass transmission, a carrier is used for transmission. This carrier is transmitted over a channel.
- If two or more bits are combined in some symbols, then the signalling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the transmission channel bandwidth. Thus because of grouping of bits in symbols, the transmission channel bandwidth is reduced.
- In quadrature phase shift keying, two successive bits in the data sequence are grouped together. This reduces the bits rate of signalling rate (i.e. f_b) and hence reduces the bandwidth of the channel.
- In BPSK we know that when symbol changes the level, the phase of the carrier is changed by 180°. Since there were only two symbols in BPSK, the phase shift occurs in two levels only.
- In QPSK two successive bits are combined. This combination of two bits forms four distinct symbols. When the symbol is changed to next symbol the

Since $b_o(t)$ and $b_e(t)$ cannot change at the same time, the phase change in QPSK signal will be maximum $\pi/2$. This is clear from Fig. 4.4.3.

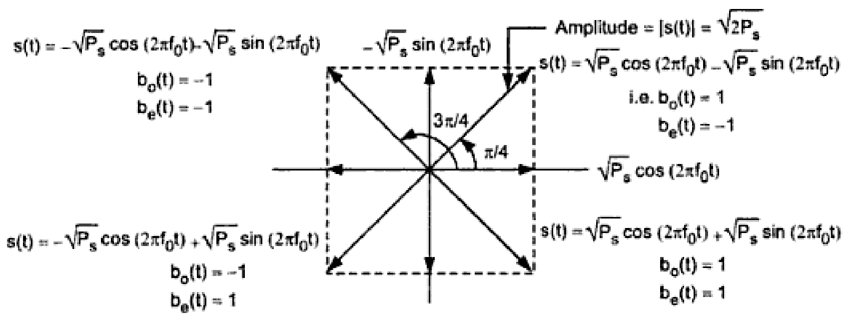


Fig. 4.4.3 Phasor diagram of QPSK signal

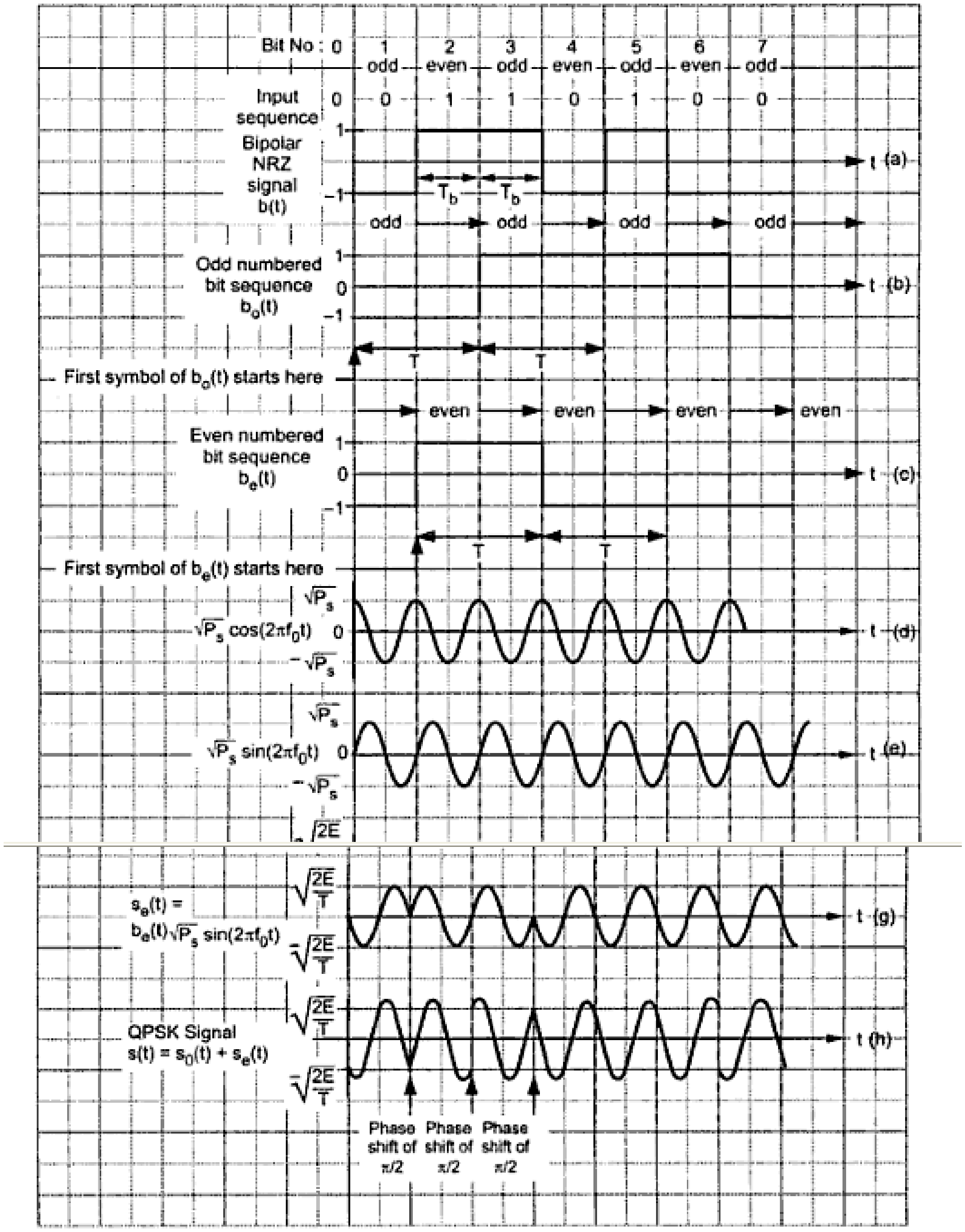


Fig. 4.4.2 QPSK waveforms (a) Input sequence and its NRZ waveform (b) Odd numbered bit sequence and its NRZ waveform (c) Even numbered bit sequence and its NRZ waveform (d) Basis function $\phi_1(t)$ (e) Basis function $\phi_2(t)$ (f) Binary PSK waveform for odd numbered channel (g) Binary PSK waveform for even numbered channel (h) Final QPSK waveform representing equation

4.4.1.3 The QPSK Receiver

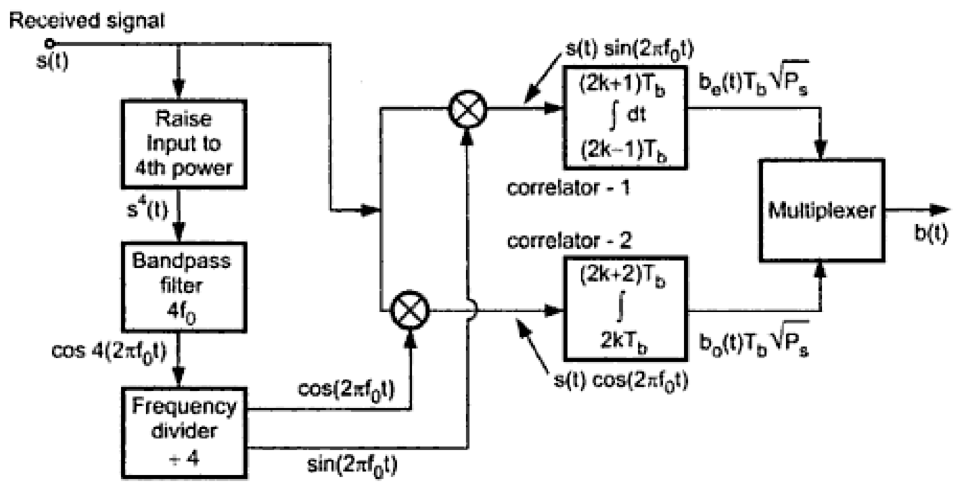


Fig. 4.4.4 QPSK receiver

Fig. 4.4.4 shows the QPSK receiver. This is synchronous reception. Therefore coherent carrier is to be recovered from the received signal $s(t)$.

Operation

Step 1 : Isolation of carrier

The received signal $s(t)$ is first raised to its 4^{th} power, i.e. $s^4(t)$. Then it is passed through a bandpass filter centered around $4f_0$. The output of the bandpass filter is a coherent carrier of frequency $4f_0$. This is divided by 4 and it gives two coherent quadrature carriers $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 (t)$.

Step 2 : Synchronous detection

These coherent carriers are applied to two synchronous demodulators. These synchronous demodulators consist of multiplier and an integrator.

Step 3 : Integration over two bits interval

The incoming signal is applied to both the multipliers. The integrator integrates the product signal over two bit interval (i.e. $T_s = 2T_b$).

Step 4 : Sampling and multiplexing odd and even bit sequences

At the end of this period, the output of integrator is sampled. The outputs of the two integrators are sampled at the offset of one bit period, T_b. Hence the output of

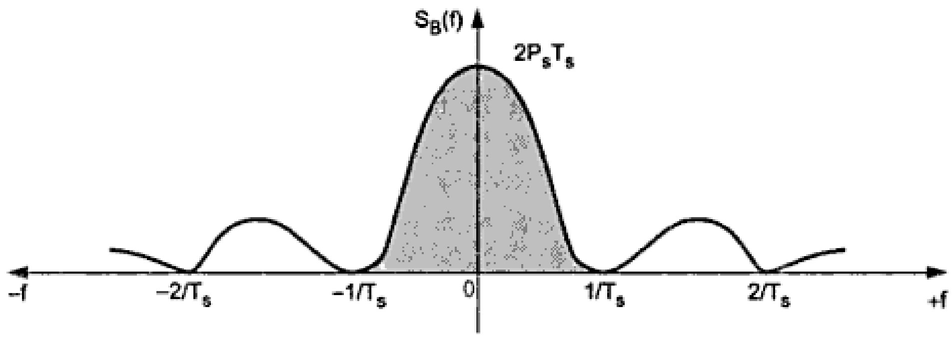


Fig. 4.4.7 Plot of power spectral density of QPSK signal

$$BW = \text{Highest frequency} - \text{Lowest frequency in main lobe}$$

$$= \frac{1}{T_s} - \left(-\frac{1}{T_s}\right) \text{ since carrier frequency } f_0 \text{ cancels out}$$

$$= \frac{2}{T_s}$$

We know that $T_s = 2T_b$

$$\therefore \quad BW = \frac{2}{2T_b} = \frac{1}{T_b} = f_b$$

which is same as we obtained in equation 4.4.24.

4.4.5 Advantages of QPSK

QPSK has some definite advantages and disadvantages as compared to BPSK and DPSK.

Advantages :

1) For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.
2) Because of reduced bandwidth, the information transmission rate of QPSK is higher.
3) Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

2.9.1 Bandwidth Efficiency (Information Density)

Definition : It is the ratio of transmission bit rate to minimum required bandwidth.

i.e.,

$$\text{BW efficiency} = \frac{\text{Transmission rate(Bits / sec)}}{\text{Minimum bandwidth (cycles / sec)}}$$

$$= \frac{\text{Transmission rate}}{\text{Minimum bandwidth}} \text{ bits/cycle} \quad ...(2.9.1)$$

- When bandwidth efficiency is normalized to 1-Hz bandwidth, it gives number of bits that can be propagated per hertz of bandwidth.
- Bandwidth efficiency is used to compare the performance of digital modulation techniques.

2.10 Carrier Synchronization (Carrier Recovery)

The carrier synchronization is required in coherent detection methods to generate a coherent reference at the receiver. In this method the data bearing signal is modulated on the carrier in such a way that the power spectrum of the modulated carrier signal contains a discrete component at the carrier frequency. That is the fourier transform of the modulated signal contains one component at f_c also. Then the phase locked loop can be used to track this component f_c. The output frequency of phase locked loop is thus locked to the carrier frequency f_c in the transmitted signal. This output frequency of phase locked loop is used as a coherent reference signal for detection in the receiver.

2.10.1 Carrier Synchronization using M^{th} Power Loop

Fig. 2.10.1 shows the block diagram of carrier recovery or carrier synchronization circuit.

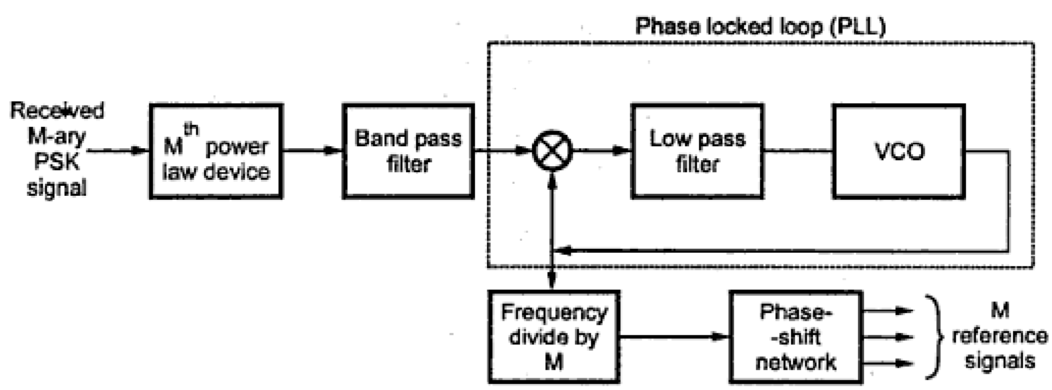


Fig. 2.10.1 Block diagram of M^{th} power loop

Fig. 2.10.1 shows the block diagram of carrier recovery circuit for M-ary PSK. This circuit is called the M^{th} power loop. When M = 2, then it is called squaring loop. When M=2, the M-ary PSK is then called as binary PSK. As shown in diagram, the input signal is first raised to the M^{th} power by the M^{th} power law device. Then the signal is passed through a bandpass filter. The bandpass filter is tuned to the carrier frequency f_c. The phase locked loop consists of a phase detector, low-pass filter and VCO. The phase locked loop tracks the carrier frequency. Then the output of a voltage controlled oscillator (VCO) is the carrier frequency. The output frequency of VCO is divided by M. This is done because the M^{th} power of the input signal multiplies carrier frequency by M. The phase shift network then separates 'M' reference signals for the 'M' correlation receivers. In this technique the power of the input signal is raised to some power say 'M'. Let us say M = 2, then the input signal is squared. Because of this, the sign of the recovered carrier is always independent of sign of the input signal carrier since it is squared. Therefore there can be 180° error in the output.

2.10.2 Costas Loop for Carrier Synchronization

May / June - 2006

This is the alternative method for carrier synchronization. This is used for binary phase shift keying. The block diagram is shown in Fig. 2.10.2.

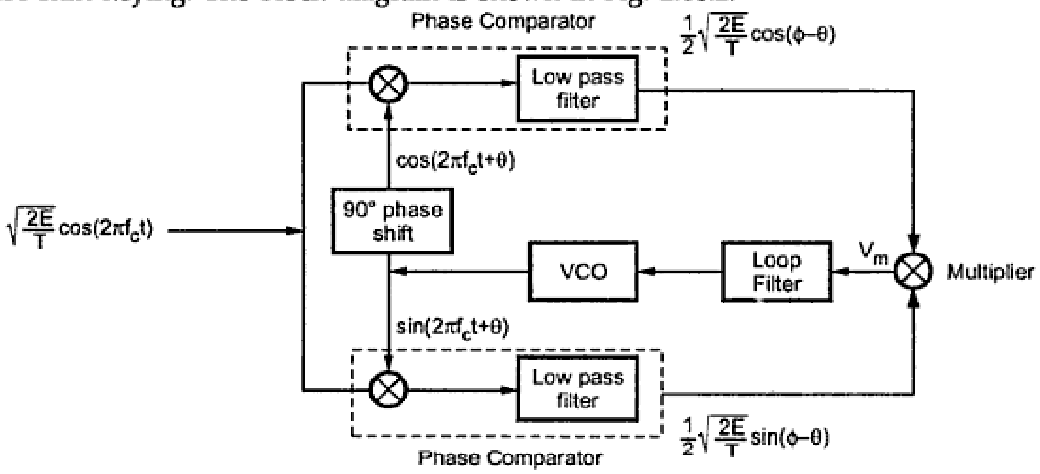


Fig. 2.10.2 The costas loop

As shown in Fig. 2.10.2 there are two phase locked loops. They have a common VCO and separate phase comparators. Let us assume that the VCO operates at the carrier frequency f_c with arbitrary phase angle θ. The BPSK signal is supplied to both the phase comparators. The low-pass filters remove the double frequency terms generated in the phase comparators and generate,

$\frac{1}{2}\sqrt{\frac{2E}{T}}\cos(\phi-\theta)$ and $\frac{1}{2}\sqrt{\frac{2E}{T}}\sin(\phi-\theta)$. The multiplier output is given as,

$$V_m = \frac{1}{4}\times\frac{2E}{T}\sin(\phi-\theta)\cos(\phi-\theta) \quad \text{... (2.10.1)}$$

$$= \frac{E}{2T}\cdot\frac{1}{2}\sin 2(\phi-\theta) \quad \text{..(2.10.2)}$$

$$= \frac{E}{4T}\sin 2(\phi-\theta) \quad \text{... (2.10.3)}$$

The power 'P' of the signal over the period T is given by,

$$P = \frac{E}{T}$$

Therefore equation (2.10.3) can be written as,

$$V_m = \frac{P}{4}\sin 2(\phi-\theta) \quad \text{... (2.10.4)}$$

If there is some difference between the VCO frequency and the input carrier frequency then the phase difference $(\phi-\theta)$ is changed proportionally. The change in $(\phi-\theta)$ causes V_m to increase or decrease VCO frequency such that synchronization is maintained.

2.5 Differential Phase Shift Keying (DPSK)

Differential phase shift keying (DPSK) is differentially coherent modulation method. DPSK does not need a synchronous (coherent) carrier at the demodulator. The input sequence of binary b its is modified such that the next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

2.5.1 DPSK Transmitter and Receiver

2.5.1.1 Transmitter / Generator of DPSK Signal

Fig. 2.5.1 shows the scheme to generate DPSK signal.

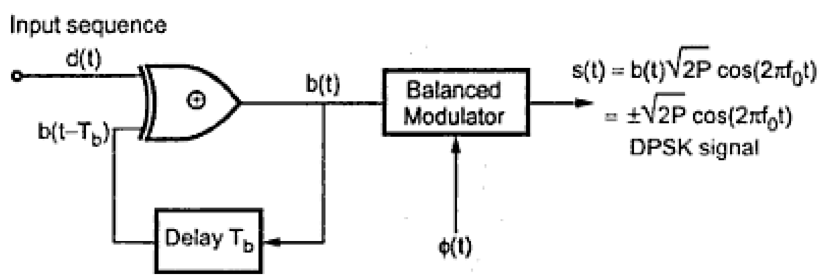


Fig. 2.5.1 Block diagram of DPSK generate or transmitter

The input sequence is $d(t)$. Output sequence is $b(t)$ and $b(t-T_b)$ is the previous output delayed by one bit period. Depending upon values of $d(t)$ and $b(t-T_b)$, exclusive OR gate generates the output sequence $b(t)$. Table 2.5.1 shows the truth table of this operation.

$d(t)$	$b(t-T_b)$	$b(t)$
0 (–1 V)	0 (–1 V)	0 (–1 V)
0 (–1 V)	1 (1 V)	1 (1 V)
1 (1 V)	0 (–1 V)	1 (1 V)
1 (1 V)	1 (1 V)	0 (–1 V)

Table 2.5.1 Truth table of exclusive OR gate

An arbitrary sequence $d(t)$ is taken. Depending on this sequence, $b(t)$ and $b(t-T_b)$ are found. These waveforms are shown in Fig. 2.5.2. The above Table 2.5.1 is used to derive the levels of these waveforms.

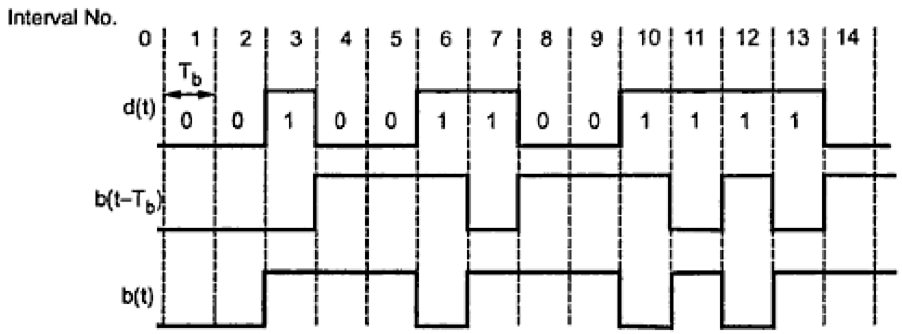


Fig. 2.5.2 DPSK waveforms

From the above waveform it is clear that $b(t-T_b)$ is the delayed version of $b(t)$ by one bit period T_b. The exclusive OR operation is satisfied in any interval i.e. in any interval $b(t)$ is given as,

$$b(t) = d(t) \oplus b(t-T_b) \quad \text{... (2.5.1)}$$

While drawing the waveforms the value of $b(t-T_b)$ is not known initially in interval no.1. Therefore it is assumed to be zero and then waveforms are drawn. We can write some important conclusions from the waveforms

1. Output sequence $b(t)$ changes level at the beginning of each interval in which $d(t)=1$ and it does not changes level when $d(t)=0$. Observe that $d(3)=1$, hence level of $b(3)$ is changed at the beginning of interval 3. Similarly in intervals 10, 11, 12 and 13 $d(t)=1$. Hence $b(t)$ is changed at the starting of these intervals. In interval 8 and 9 $d(t)=0$. Hence $b(t)$ is not changed in these intervals.
2. When $d(t)=0$, $b(t)=b(t-T_b)$ and
   When $d(t)=1$, $b(t)=\overline{b(t-T_b)}$
3. In interval no.1. we have assumed $b(t-T_b)=0$ and we obtained the waveform as shown in Fig. 2.5.2. If we assume $b(t-T_b)=1$ in interval no. 1, then the waveform of $b(t)$ will be inverted. But still $b(t)$ changes the level at the beginning of each interval in which $d(t)=1$.
4. The sequence $b(t)$ modulates sinusoidal carrier.
5. When $b(t)$ changes the level, phase of the carrier is changed. Since $b(t)$ changes its level only if $d(t)=1$; It shows that phase of the carrier is changed only if $d(t)=1$. In PSK phase of the carrier changes on both the symbol '1' and '0'. Whereas in DPSK phase of the carrier changes only on symbol '1'. This is the main difference between PSK and DPSK.
6. Always two successive bits of $d(t)$ are checked for any change of level. Hence one symbol has two bits.

$\therefore$ Symbol duration (T) = Duration of two bits $(2T_b)$

i.e.
$$T = 2T_b \quad \text{... (2.5.2)}$$

As shown in Fig. 2.5.1, the sequence $b(t)$ is applied to a balanced modulator. The balanced modulator is also supplied with a carrier $\sqrt{2P}\cos(2\pi f_0 t)$

The modulator output is,

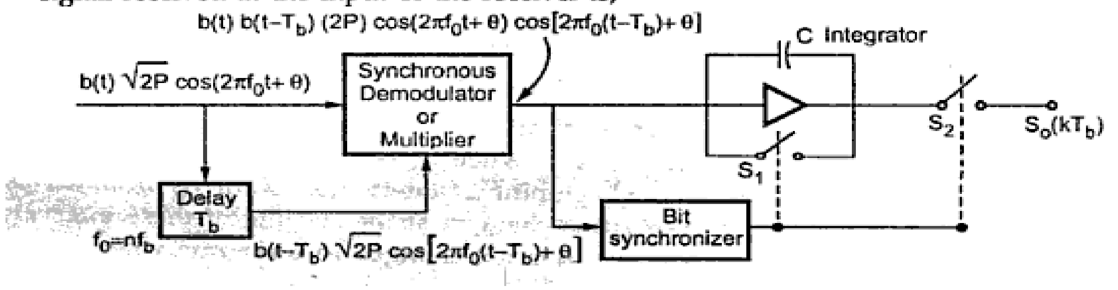
$$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t) \quad \text{... (2.5.3)}$$

$$= \pm\sqrt{2P}\cos(2\pi f_0 t) \quad \text{... (2.5.4)}$$

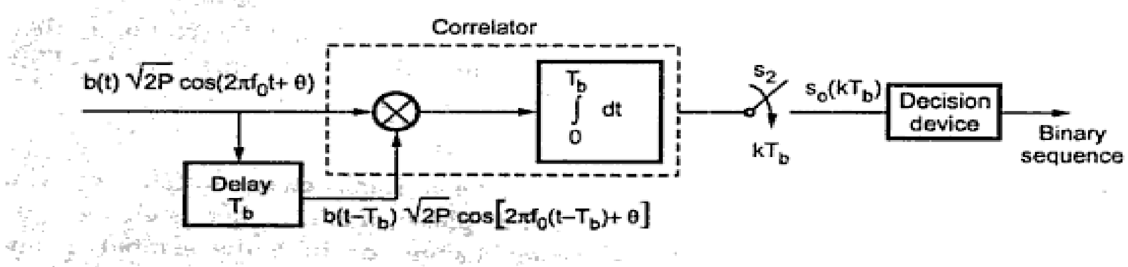
The above equation gives DPSK signal. Fig. 2.5.2 shows this DPSK waveforms. As shown in the waveforms the phase changes only when $d(t) = 1$.

2.5.1.2 DPSK Receiver

Fig. 2.5.3 shows the method to recover the binary sequence from DPSK signal. Fig. 2.5.3 (a) and (b) are equivalent to each other. Fig. 2.5.3(b) represents DPSK receiver using correlator. Fig. 2.5.3(a) shows multiplier and integrators separately. During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the input of the receiver is,



(a)



(b)

Received signal $= b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$ ... (2.5.5)

This signal is multiplied with its delayed version by one bit. Therefore the outpu of the multiplier is,

Multiplier output $= b(t)\, b(t-T_b)\,(2P)\cos(2\pi f_0 t+\theta)\cos[2\pi f_0(t-T_b)+\theta]$ ... (2.5.6)

We know that, $\cos(A)\cos(B) = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$

Here $A = 2\pi f_0 t + \theta$ and $B = 2\pi f_0 (t-T_b)+\theta$

$\therefore$ Multiplier output $= b(t)\, b(t-T_b)\, P\left\{\cos 2\pi f_0 T_b + \cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]\right\}$ ...(2.5.7)

f_0 is the carrier frequency and T_b is one bit period. T_b contains integral number o cycles of f_0. We know that,

$$f_b = \frac{1}{T_b}$$

If T_b contains 'n' cycles of f_0 then we can write,

$$f_0 = n f_b \quad \Rightarrow \quad f_0 = \frac{n}{T_b}$$

$\therefore$ $$f_0 T_b = n$$ ... (2.5.8)

Putting $f_0 T_b = n$ in first cosine term in equation (2.5.7) we get

$$\text{Multiplier output} = b(t)\, b(t-T_b)\, P\left\{\cos 2\pi n + \cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right]\right\}$$

This signal is given to the integrator. In the k^{th} bit interval, the integrator output can be written as,

$$s_o(kT_b) = b(kT_b)\, b\left[(k-1)T_b\right] P \int_{(k-1)T_b}^{kT_b} dt + b(kT_b)\, b\left[(k-1)T_b\right] P \int_{(k-1)T_b}^{kT_b} \cos\left[4\pi f_0\left(t-\frac{T_b}{2}\right)+2\theta\right] dt$$

The integration of the second term will be zero since it is integration of carrier over one bit duration. The carrier has integral number of cycles over one bit period hence integration is zero. Therefore we can write,

$$\begin{aligned} s_o(kT_b) &= b(kT_b)\, b\left[(k-1)T_b\right] P\left[kT_b-(k-1)T_b\right] \\ &= b(kT_b)\, b\left[(k-1)T_b\right] PT_b \qquad \text{... (2.5.10)} \end{aligned}$$

Here know that $PT_b = E_b$; i.e. energy of one bit. The product $b(kT_b)\, b\left[(k-1)T_b\right]$ decides the sign of PT_b.

The transmitted data bit $d(t)$ can be verified easily from product $b(kT_b)\, b\left[(k-1)T_b\right]$. We know from Fig. 2.5.2 when $b(t) = b(t-T_b)$, $d(t) = 0$. That is if both are $+1V$ or $-1V$ then $b(t)\, b(t-T_b) = 1$. Alternately we can write,

If $\quad b(t)\, b(t-T_b) = 1V \quad$ then $\quad d(t) = 0$

We know that $b(t) = \overline{b(t-T_b)}$ then $d(t) = 1$. That is $b(t) = -1\,V, b(t-T_b) = +1\,V$ and vice versa. Therefore $b(t)\, b(t-T_b) = -1$. Alternately we can write,

If $\quad b(t)\, b(t-T_b) = -1V, \quad$ then $\quad d(t) = 1$

The decision device is shown in Fig. 2.5.3 (b). We know that,

$$s_o(kT_b) = b(kT_b)\, b\left[(k-1)T_b\right] PT_b \qquad \text{... from equation 2.5.10}$$

If $\quad s_o(kT_b) = \begin{cases} -PT_b, \text{ then } d(t) = 1 \text{ and} \\ +PT_b, \text{ then } d(t) = 0 \end{cases}$

UNIT III DIGITAL TRANSMISSION

INTRODUCTION:

3.1 Pulse Modulation

The continuous time signal $x(t)$ to be transmitted is sampled at frequency f_s sufficiently above the highest frequen cy present in $x(t)$. The amplitude of the modulating signal $x(t)$ modulates some parameter of the pulse train. These parameters are amplitude, duration (width) and position. Fig. 3.1.1 shows different types of analog pulse modulation techniques with message waveform $x(t)$.

For PAM the modulated pulse parameter is amplitude, for PDM it is width and for PPM it is relative position. These parameters vary in direct proportion to amplitude of $x(t)$ at the sampling instant. As shown in waveforms of Fig. 3.1.1.

$$f_s = \frac{1}{T_s} = \text{Sampling frequency}$$

$$A_0 = \text{Amplitude of the pulse}$$

and τ_0 = Width of the pulse

Since the waveforms are unipolar, they have some dc value. Also the shape of the pulse should be preserved (rising and falling edges, amplitude, duration etc.). Thus the transmission bandwidth needed for these pulse transmission is quite high compared to the message signal bandwidth. Therefore normally single channel PAM, PPM or PDM are seldom used. Always time division multiplexing (TDM) is used to utilize the channel bandwidth.

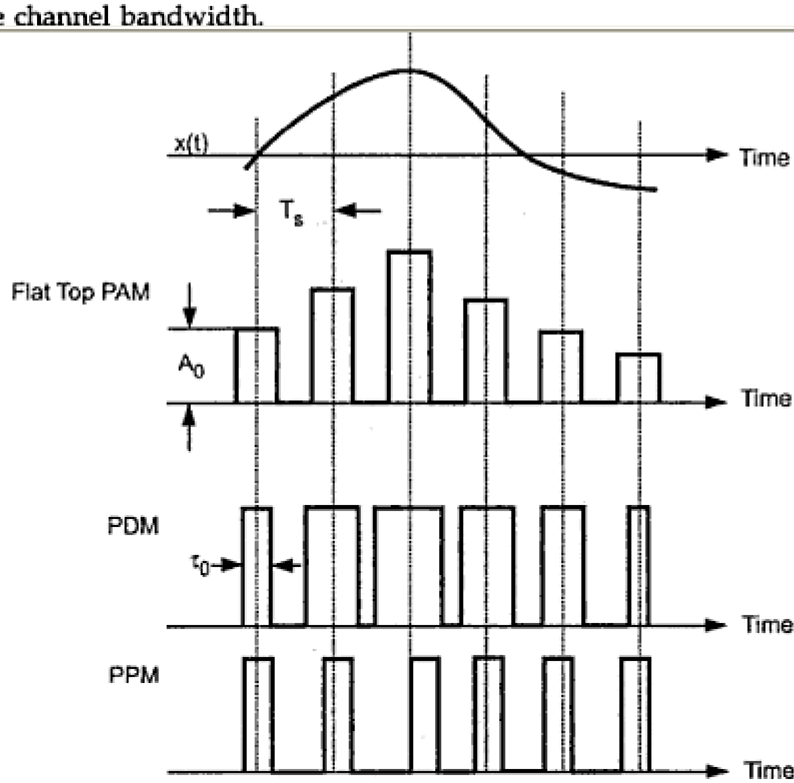


Fig. 3.1.1 Different types of analog pulse modulation techniques

3.2 Pulse Code Modulation (PCM)

3.2.1 PCM Generator

Nov./Dec.-2005 ; May/June - 2006

The pulse code modulator technique samples the input signal $x(t)$ at frequency $f_s \geq 2W$. This sampled 'Variable amplitude' pulse is then digitized by the analog to digital converter. The parallel bits obtained are converted to a serial bit stream. Fig. 3.2.1 shows the PCM generator.

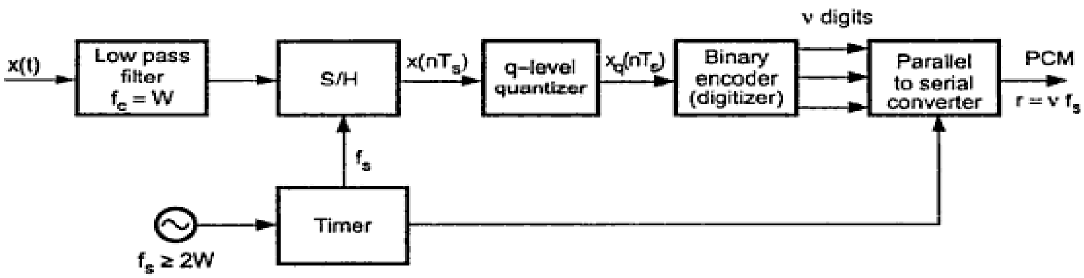


Fig. 3.2.1 PCM generator

In the PCM generator of above figure, the signal $x(t)$ is first passed through the low-pass filter of cutoff frequency 'W' Hz. This low-pass filter blocks all the frequency components above 'W' Hz. Thus $x(t)$ is bandlimited to 'W' Hz. The sample and hold circuit then samples this signal at the rate of f_s. Sampling frequency f_s is selected sufficiently above Nyquist rate to avoid aliasing i.e.,

$$f_s \geq 2W$$

In Fig. 3.2.1 output of sample and hold is called $x(nT_s)$. This $x(nT_s)$ is discrete in time and continuous in amplitude. A q-level quantizer compares input $x(nT_s)$ with its fixed digital levels. It then assigns any one of the digital level to $x(nT_s)$ which results in minimum distortion or error. This error is called *quantization error*. Thus output of quantizer is a digital level called $x_q(nT_s)$.

Quantization error is given as,

$$\varepsilon = x_q(nT_s) - x(nT_s) \quad \text{... (3.2.1)}$$

3.2.2 Transmission Bandwidth in PCM

Let the quantizer use 'v' number of binary digits to represent each level. Then the number of levels that can be represented by 'v' digits will be,

$$q = 2^v \quad \text{... (3.2.2)}$$

Here 'q' represents total number of digital levels of q-level quantizer.

For example if $v = 3$ bits, then total number of levels will be,

$$q = 2^3 = 8 \text{ levels}$$

Each sample is converted to 'v' binary bits. i.e. Number of bits per sample = v

We know that, Number of samples per second = f_s

∴ Number of bits per second is given by,

$$\begin{aligned}(\text{Number of bits per second}) &= (\text{Number of bits per samples}) \\ &\quad \times (\text{Number of samples per second}) \\ &= v \text{ bits per sample} \times f_s \text{ samples per second}\end{aligned}$$

3.2.3 PCM Receiver

Nov./Dec.-2005 ; May/June - 2006

Fig. 3.2.2 (a) shows the block diagram of PCM receiver and Fig. 3.2.2 (b) shows the reconstructed signal. The regenerator at the start of PCM receiver reshapes the pulses and removes the noise. This signal is then converted to parallel digital words for each sample.

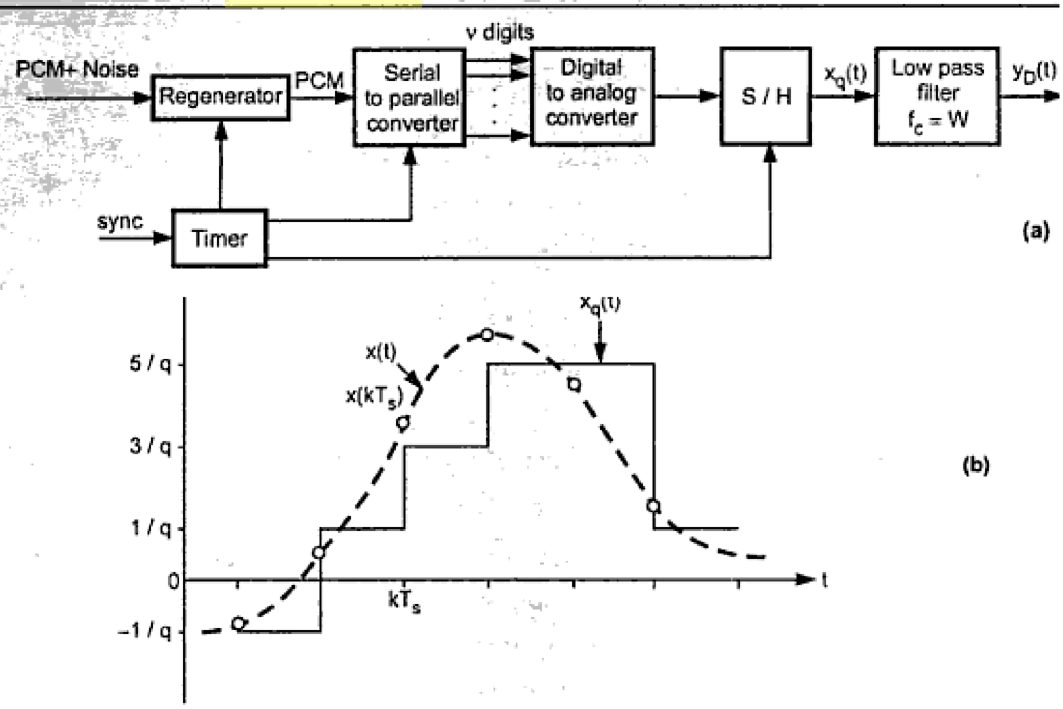


Fig. 3.2.2 (a) PCM receiver
(b) Reconstructed waveform

The digital word is converted to its analog value $x_q(t)$ along with sample and hold. This signal, at the output of S/H is passed through lowpass reconstruction filter to get $y_D(t)$. As shown in reconstructed signal of Fig. 3.2.2 (b), it is impossible to reconstruct exact original signal $x(t)$ because of permanent quantization error introduced during quantization at the transmitter. This quantization error can be reduced by increasing the binary levels. This is equivalent to increasing binary digits (bits) per sample. But increasing bits 'v' increases the signaling rate as well as transmission bandwidth as we have seen in equation (3.2.4) and equation (3.2.7).

1.8.6.3 Companding in PCM

Normally we don't know how the signal level will vary in advance. Therefore the nonuniform quantization (variable step size 'δ') becomes difficult to implement. Therefore the signal is amplified at low signal levels and attenuated at high signal levels. After this process, uniform quantization is used. This is equivalent to more step size at low signal levels and small step size at high signal levels. At the receiver a reverse process is done. That is signal is attenuated at low signal levels and amplified at high signal levels to get original signal. Thus the compression of signal at transmitter and expansion at receiver is called combinely as *companding*. Fig. 1.8.9 shows compression and expansion curves.

As can be seen from Fig. 1.8.9, at the receiver, the signal is expanded exactly opposite to compression curve at transmitter to get original signal. A dotted line in the Fig. 1.8.9 shows uniform quantization. The compression and expansion is obtained by passing the signal through the amplifier having nonlinear transfer characteristic as

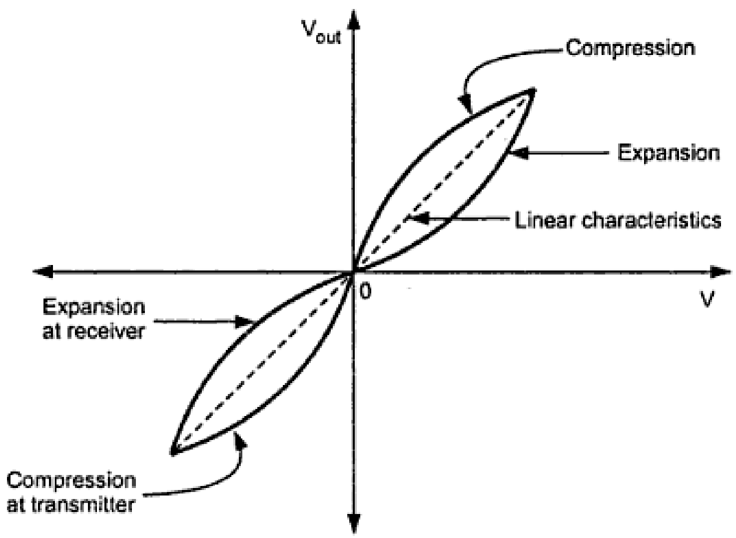


Fig. 1.8.9 Companding curves for PCM

1.8.6.4 μ - Law Companding for Speech Signals

Normally for speech and music signals a μ - law compression is used. This compression is defined by the following equation,

$$Z(x) = (\text{Sgn } x) \frac{ln(1+\mu|x|)}{ln(1+\mu)} \quad |x| \leq 1 \qquad \ldots (1.8.52)$$

Fig. 1.8.10 shows the variation of signal to noise ratio with respect to signal level without companding and with companding.

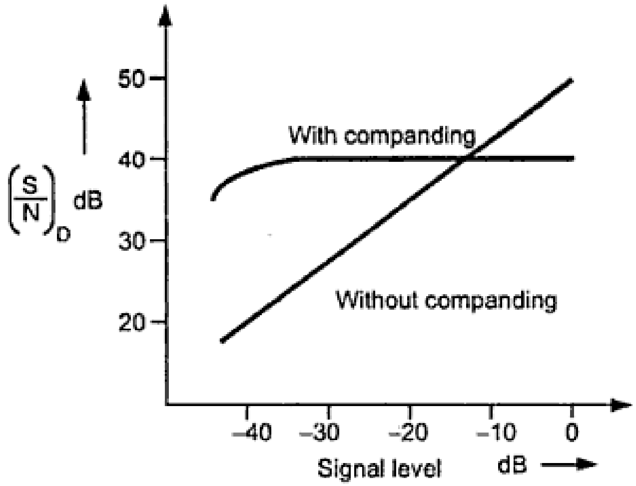


Fig. 1.8.10 PCM performance with μ - law companding

1.8.6.5 A-Law for Companding

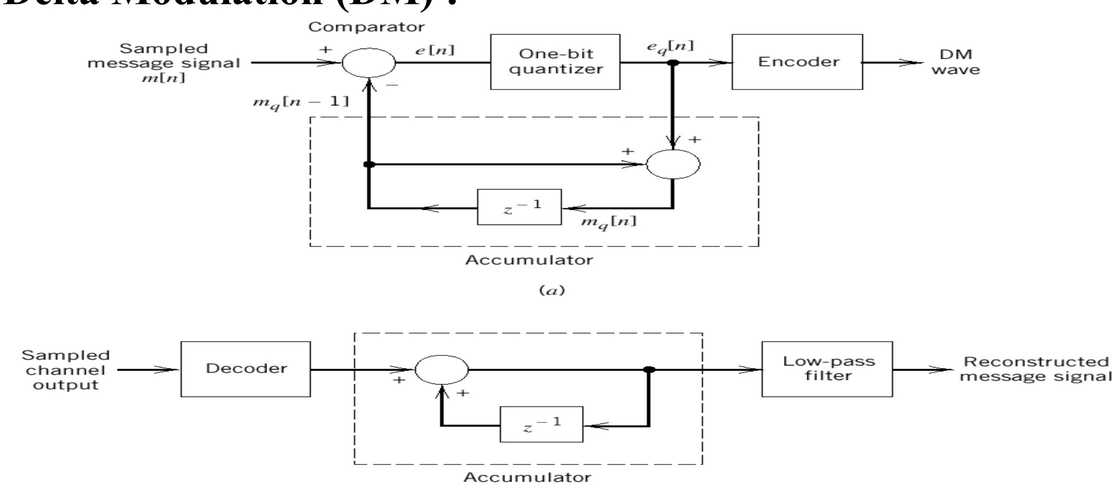
The A law provides piecewise compressor characteristic. It has linear segment for low level inputs and logarithmic segment for high level inputs. It is defined as,

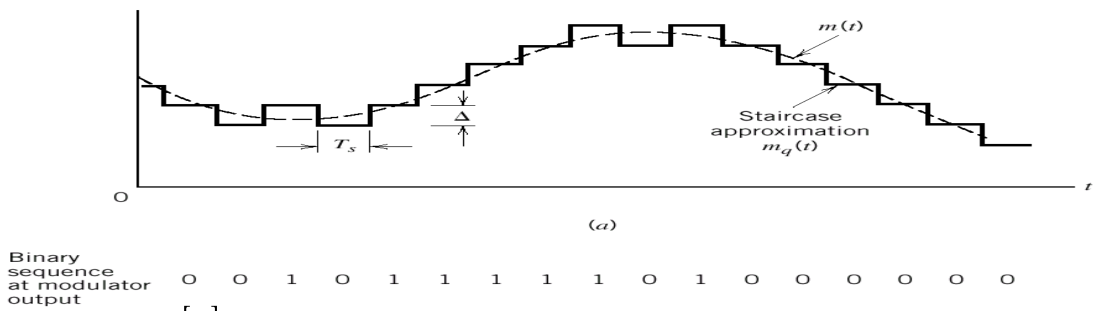
$$Z(x) = \begin{cases} \dfrac{A|x|}{1+\ln A} & \text{for } 0 \le |x| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|x|)}{1+\ln A} & \text{for } \dfrac{1}{A} \le |x| \le 1 \end{cases} \quad \ldots (1.8.53)$$

When A = 1, we get uniform quantization. The practical value for A is 87.56. Both A-law and μ-law companding is used for PCM telephone systems.

Delta Modulation (DM) :

Delta Modulation (DM) :





Let $m[n] = m(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$

where T_s is the sampling period and $m(nT_s)$ is a sample of $m(t)$.

The error signal is

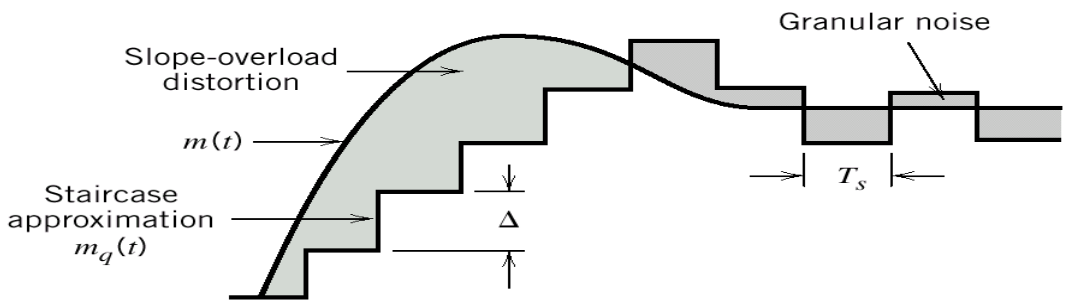
$e[n] = m[n] - m_q[n-1]$ (3.52)

$e_q[n] = \Delta \operatorname{sgn}(e[n])$ (3.53)

$m_q[n] = m_q[n-1] + e_q[n]$ (3.54)

where $m_q[n]$ is the quantizer output , $e_q[n]$ is the quantized version of $e[n]$, and Δ is the step size

The modulator consists of a comparator, a quantizer, and an accumulator. The output of the accumulator is



Differential Pulse-Code Modulation (DPCM):

Usually **PCM** has the sampling rate higher than the **Nyquist rate** .The encode signal contains redundant information. **DPCM** can efficiently remove this redundancy**.**

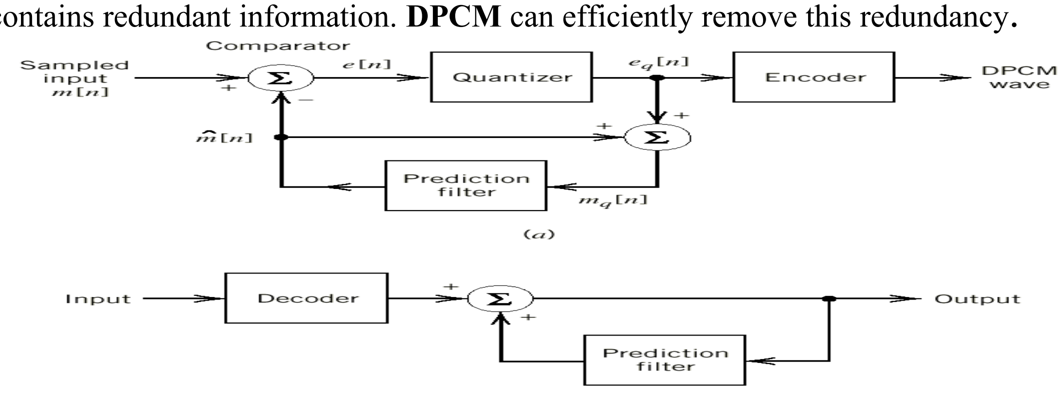


Figure 3.28 DPCM system. (*a*) Transmitter. (*b*) Receiver.

$e[n] = m[n] - \hat{m}[n]$ (3.74)

$\hat{m}[n]$ is a prediction value.

The quantizer output is

$e_q[n] = e[n] + q[n]$ (3.75)

where $q[n]$ is quantization error.

The prediction filter input is

$$m_q[n] = \underbrace{\hat{m}[n] + e[n]}_{m[n]} + q[n] \quad (3.77)$$

$$\Rightarrow m_q[n] = m[n] + q[n] \quad (3.78)$$

Processing Gain:

The (SNR) $_o$ of the DPCM system is

$$(\text{SNR})_o = \frac{\sigma_M^2}{\sigma_Q^2} \quad (3.79)$$

where σ_M^2 and σ_Q^2 are variances of $m[n]$ $(E[m[n]] = 0)$ and $q[n]$

$$(\text{SNR})_o = \left(\frac{\sigma_M^2}{\sigma_E^2}\right)\left(\frac{\sigma_E^2}{\sigma_Q^2}\right)$$

$$= G_p (\text{SNR})_Q \quad (3.80)$$

where σ_E^2 is the variance of the predictions error

and the signal-to-quantization noise ratio is

$$(\text{SNR})_Q = \frac{\sigma_E^2}{\sigma_Q^2} \quad (3.81)$$

$$\text{Processing Gain, } G_p = \frac{\sigma_M^2}{\sigma_E^2} \quad (3.82)$$

Design a prediction filter to maximize G_p (minimize σ_E^2)